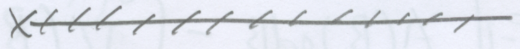
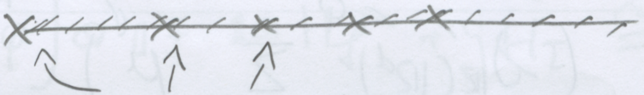
RIMS. 2014/10/9

① QM + QFT $\quad H = H_{Sch} \otimes 1 + 1 \otimes H_{field} + gH_I$

$\sigma(H_{Sch}) = \{E_j\} \cup (0, \infty) =$ 

$\sigma(H_{field}) = [0, \infty) =$ 

$\sigma(H_{g=0}) =$ 

Embedded e v

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② H_{Sch}

Ex $H_{Sch} = -\frac{1}{2}\Delta + V \quad (f, e^{-tH} g) = \int dx\, \mathbb{E}_P^x\left[f(B_0) g(B_t) e^{-\int_0^t V} \right]$

Ex $H_{Sch} \varphi_p = E_p \varphi_p, \quad E_p = \inf \sigma(H_{Sch}) \quad$ g.s.

$\boxed{\varphi_p > 0} \quad U_g \; L^2(\mathbb{R}^d, dx) \longrightarrow L^2(\mathbb{R}^d, \varphi_p^2 dx)$

$\qquad \overset{\psi}{f} \qquad \longmapsto \qquad \overset{\psi}{\frac{1}{\varphi} f} \quad$ unitary

$\frac{1}{\varphi}(H - E)\varphi \cdot L$

V is kato class potential $\Longrightarrow \exists\, P(\phi)_1$-process $(X_t)_{t \in \mathbb{R}}$

$(f, e^{-tL} g)_{L^2(\mathbb{R}^d, \varphi_p^2 dx)} = \int dx\, \mathbb{E}_N^x [f(X_0) g(X_t)]$

Ex $H = \frac{1}{2}(p \quad A)^2 + V \qquad S := -i\int_0^t A\, dB_s$

$(f, e^{tH} g) = \int dx\, \mathbb{E}_P^x\left[\overline{f(B_0)} g(B_t) e^{S} \right] \quad -\int_0^t V(B_s) ds$

Ex $\sqrt{(p-A)^2 + m^2} \quad m + V = \Psi\left(\frac{1}{2}(p-A)^2\right) + V$

$\Psi(u) = \sqrt{2u + m^2} \quad$ in $\quad \mathcal{B} \Longleftrightarrow \mathbb{R}^+ \times \mathcal{L} \Longleftrightarrow \mathcal{S}$

$\qquad\qquad\qquad\qquad\qquad (b, \lambda) \qquad\qquad (T_t)$

$\mathcal{B} \ni \Psi(u) = bu + \int_0^t 1 - e^{-uy} d\lambda(y)$

$\mathbb{E}\left[e^{-uT_t} \right] = e^{-t\Psi(u)} \quad \Big/ \quad (f, e^{t\Psi(H)} g) = \mathbb{E}\left[(f, e^{T_t H} g) \right]$

④ Nelson model Scalar field + particle

$\tilde{H} = H_{Sch} \otimes 1 + 1 \otimes H_f + g H_I$ on $L^2(\mathbb{R}^d) \otimes \mathcal{F}$

$H_{Sch} = -\frac{1}{2}\Delta_x + V(x)$ $H_I = \phi(\varrho(\cdot - x))$

$H = \left[\frac{1}{\varphi}(H_{Sch} - E)\varphi\right] \otimes 1 + 1 \otimes H_f + g H_I \cong \hat{H}$

on $L^2(\mathbb{R}^d \times Q, \varphi_p^2 dx \otimes d\mu) = \mathcal{H}$

$$(F, e^{-tH} G)_{\mathcal{H}} = \int \varphi^2(x) dx\, \mathbb{E}_p^x \left[J_0 F(X_0) \cdot J_t^* G(X_t) \cdot e^{-g\int_0^t \phi_E(j_s \varphi(\cdot - X_s))ds} \right]$$

∵ Trotter product formula + $J_t^* J_s = e^{-(t-s)H_f}$

⑤ Spectral analysis

Ground state? Gibbs meas, UV renormalization

$\varrho(\cdot - x) \to \delta(\cdot\; x)$ or $\hat{\varphi}(k) e^{-ikx} \to 1\, e^{-ikx}$

$\boxed{GS}$ $\underset{\varphi_g^t}{\underset{\shortparallel}{e^{-tH} 1}} / \| e^{tH} 1 \| \to \varphi_g$ $(1, \varphi_g^t)^2 = \dfrac{(1, e^{tH} 1)^2}{(1, e^{-2tH} 1)} = \gamma(t)$

$$\gamma(t) \to a \begin{cases} > 0 & \exists gs \\ = 0 & \nexists gs \end{cases}$$

$$(1, e^{tH} 1) = \int \varphi^2(x) dx\, \mathbb{E}_p^x \left[e^{+\frac{g^2}{4} \left\| \int_0^t j_s \varphi(\cdot - X_s) ds \right\|^2} \right]$$

$$\int_0^T dt \int_0^T ds \underbrace{\int e^{-|t-s|\omega} |\hat{\varphi}|^2 e^{-ik(X_t - X_s)} dk}_{W(X_t - X_s,\, t-s)}$$ double potential

Spohn $\int \frac{|\hat{\varphi}|^2}{\omega^3} < \infty \Rightarrow \exists gs$

LMS $\int \frac{|\hat{\varphi}|^2}{\omega^3} = \infty \Rightarrow \nexists g.s.$

$H_\Psi = \Psi(\frac{1}{2}(p\ A)^2) + V$

$(f, e^{-tH_\Psi} g) = \int dx\, \mathbb{E}^{x,0}_{P\times\nu}\left[\bar{f}(B_{T_0})\, g(B_{T_t})\, e^{S_\Psi}\right]$

$S_\Psi = -i\int_0^{T_t} A(B_s)\circ dB_s - \int_0^t V(B_{T_s})\, ds$

Gaussian meas

③ $H_f$eld $\quad (\overset{Q}{\mathcal{S}'_{\mathbb{R}}(\mathbb{R}^d)}, \Sigma, \mu)$

$\langle \phi, f\rangle = \phi(f) \quad \mathbb{E}_\mu[\phi(f)] = 0, \quad \mathbb{E}_\mu[\phi(f)\phi(g)] = \frac{1}{2}(f\ g)$

Extention $\underline{\phi(f)\quad f\in L^2(\mathbb{R}^d)}$

$\mathbb{E}_\mu[e^{i\phi(f)}] = e^{-\frac{1}{4}\|f\|^2}$

n-particle space

$(:\prod_j^m \phi(f_j):, :\prod_i^n \phi(g_i):) = 0$ if $m \neq n$ $\quad L^2(Q,\mu) = \sum L^2_n(Q\mu)$

$= \mathcal{F}$

$A: L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^d) \quad \|A\| \le$

$\Gamma(A) :\prod \phi(f_j): = :\prod \phi(Af_j): \quad \Gamma(A)\mathbb{1} = \mathbb{1}$

$\Gamma(A)\Gamma(B) = \Gamma(AB), \quad \Gamma(A)^* = \Gamma(A^*) \quad \hat{\omega} = \sqrt{-\Delta + m^2}$

$\Gamma(e^{it\omega}) = e^{-it\, d\Gamma(\omega)} \quad d\Gamma(\omega) = H_{field} = H_f$ field H

$\omega \to \mathbb{1} \quad d\Gamma(\mathbb{1}) = N \quad N :\prod^n \phi(f_j): = n :\prod^n \phi(f_j):$

$(\mathcal{S}'_{\mathbb{R}}(\mathbb{R}^{d+1}), \Sigma_E, \mu_E) \quad \exists\, j_t: L^2(\mathbb{R}^d) \to L^2(\mathbb{R}^{d+1})$

Q_E

s.t. $j_t^* j_s = e^{-|t-s|\omega} \quad \Gamma(j_t) = J_t: L^2(Q) \to L^2(Q_E)$

$J_t^* J_s = e^{-|t-s|H_f}, \quad H_f\mathbb{1} = 0,$

$k \in \mathbb{R}$

Bernstein

$(\Psi, N^m\Psi) \sim (\Psi, N^k\Psi) = (\Psi, N^{m+\alpha}\Psi) = (\Psi, N^m\underline{\Psi}(N)\Psi)$

$\int(\Psi, N^m(1-e^{-\beta N})\Psi)\, d\lambda(\beta)$

$$(\varphi_g, e^{-\beta N}\varphi_g) = (e^{-tH}\mathbb{1}, e^{-\beta N} e^{-tH}\mathbb{1}) \Big/ (\mathbb{1}, e^{-2tH}\mathbb{1})$$

$$= \frac{\int \varphi^2 dx\, \mathbb{E}_p^x\left[e^{(1-e^{-\beta})\int_{-T}^0 dt \int_0^T ds\, W(X_t - X_s, t-s) + \int_{-T}^T\int_{-T}^T W}\right]}{\int \varphi^2 dx\, \mathbb{E}_p^x\left[e^{+\int_{-T}^T\int_{-T}^T W}\right]}$$

$$= \mathbb{E}_{\mu_T}\left[e^{(1-e^{-\beta})\int_{-T}^0\int_0^T W}\right]$$

Thm H $\mu_\infty \overset{\exists}{\to} \mu$ local weak conv

$$(\varphi_g, e^{\beta N}\varphi_g) = \mathbb{E}_{\mu_\infty}\left[e^{-(1-e^{-\beta})\int_{-\infty}^0\int_0^\infty W}\right) \xRightarrow{\text{analytic cont}} (\varphi_g, e^{+\beta N}\varphi_0) < \infty$$

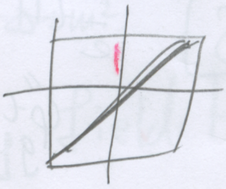
$\forall \beta \in \mathbb{R}$

$$(\varphi_g, e^{i\beta\phi(f)}\varphi_g) = e^{-\frac{\beta^2}{4}\|f\|^2}\,\mathbb{E}_{\mu_\infty}\left[e^{i\beta K_f}\right]$$

Hence $(\varphi_g, e^{-\beta\phi(f)^2}\varphi_g) = (2\pi)^{-\frac{1}{2}}\int e^{-k^2/2}(\varphi_g, e^{ik\sqrt{2\beta}\phi(f)}\varphi_g)\,dk$

$$= \frac{1}{\sqrt{1+\beta\|f\|^2}}\,\mathbb{E}_{\mu_\infty}\left[e^{-\frac{\beta K_f^2}{1+\beta\|f\|^2}}\right] \Longrightarrow \text{analytic cont}$$

$$(\mathbb{1}, e^{2TH}\mathbb{1}) = \int dx\, \varphi_p(x)^2\, \mathbb{E}_p^x\left[e^{\frac{g^2}{2}\int_{-T}^T\int_{-T}^T W}\right]$$



$\hat{\varphi} \to e^{\varepsilon|k|^2/2}$ $\quad \varepsilon \downarrow 0$

$H_\varepsilon \to ?$

$$E_\varepsilon = \int_{\mathbb{R}^3} e^{-\varepsilon|k|^2}\frac{1}{|k|^2/2+\omega|k|}\,dk$$

$$(\mathbb{1}, e^{2T(H_\varepsilon - E_\varepsilon)}\mathbb{1}) = \int dx\,\varphi_p^2\,\mathbb{E}_p^x\left[e^{\frac{g^2}{2}\int_{-T}^T\int_{-T}^T (W-E)}\right] \xrightarrow{\text{conv}}$$

$\frac{1}{2}(P-A)^2 + V + H_f$ / $\sigma_z\otimes 1 + \sigma_x\otimes\phi(f) + H_f$ /

$\sqrt{(P-A)^2+m^2} - m + V + H_f$ /